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Rayleigh Model for Electromagnetic Scattering from a Chiral Sphere

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Abstract

Rayleigh model has been proposed to deal with scattering from a small sized chiral sphere. A chiral sphere in Rayleigh size illuminated by electromagnetic waves can be substituted by four Hertzian dipoles. Thus the scattering problem is converted to a radiation problem. The four equivalent dipole moments are solved by applying the boundary conditions of Maxwell equations. Numerical results calculated by the model are compared with analytical solutions and achieve good agreement. Scattering patterns of a small chiral sphere with different chirality parameters are depicted and the effects of the chirality are discussed.

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Keywords: Electromagnetic scattering; Rayleigh model; Chiral sphere.

1. Introduction

Chiral material was first found as optical active media which rotates polarization plane of light traveling through it. Due to its special constitutive relations, chiral medium has unusual electromagnetic characteristic and is widely applied in many fields [1]. Substances with handed microstructures, such as solutions of grape sugar, tartaric acid and some biological particles, can be regarded as chiral media. Scattering from chiral particles is an important aspect in particle technology. The analytical solution of electromagnetic scattering from a chiral sphere was first presented by Bohren in 1974 [2]. Based on his work, the authors have improved the algorithms and made the calculation

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possible for large chiral spheres [3]. For scattering from chiral particles with size much smaller than the wavelength, the problem can be solved by quasi-static field analysis [4], which is similar to the idea of Rayleigh scattering.

This paper presents the research on Rayleigh scattering from a chiral sphere. For Rayleigh scattering of an ordinary dielectric particle, the particle can be regarded as a Hertzian dipole. The authors try to give a similar physical figure to describe a chiral particle. The Rayleigh sized chiral sphere illuminated by electromagnetic waves is equivalent to four Hertzian dipoles: two Hertzian electric dipoles placed perpendicularly to each and two Hertzian magnetic dipoles placed perpendicularly. Therefore, the scattered field of the chiral sphere is the total radiation fields of the four dipoles. In the following theory section, the idea is fully explained and the equivalent dipoles moments are derived by applying the boundary conditions of Maxwell equations. In Section 3, scattering pattern of a small chiral sphere is depicted according to the numerical results calculated by the model. In the whole paper, a time dependence of $\exp(-i\omega t)$ is assumed.

2. Theoretical formulations

Section 2.1 presents the ideas of Rayleigh model for electromagnetic scattering from a chiral sphere, taking account of the constitutive relations of chiral media and the theory of Rayleigh scattering from an isotropic sphere. In section 2.2, the four equivalent dipoles moments introduced in section 2.1 are solved, along with the relations between the scattered fields and the incident fields.

2.1. Rayleigh scattering from a chiral sphere.

In electromagnetics, a medium is described by its constitutive relations. For chiral material, the relation is:

$$\mathbf{D} = \varepsilon_c \mathbf{E} + i\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{H}, \quad \mathbf{B} = -i\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{E} + \mu_c\mathbf{H}, \tag{1}$$

where ε_c, μ_c is the permittivity and permeability of the medium, respectively; ε_0, μ_0 is the permittivity and permeability of vacuum, respectively; and κ is the chirality parameter. It can be seen that, in a chiral medium, the electric displacement $\mathbf{D}$ and the magnetic induction $\mathbf{B}$ depend on both electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$. This means that, differently from the usual isotropic medium, the electric field $\mathbf{E}$ causes not only induced electric moments, but also induced magnetic moments in the medium; and similarly magnetic field $\mathbf{H}$ induces not only the magnetic moments, but also the electric moments.

For Rayleigh scattering from a chiral sphere whose radius is much smaller than the incident wavelength, the incident electric field and magnetic field surrounding the sphere are nearly static fields. Due to the special polarization and magnetization presented above, the sphere can be equivalent to four Hertzian dipoles: two electric dipoles respectively placed along the directions of incident electric field and magnetic field, and two magnetic dipoles placed in the same way. Consider a small chiral sphere located at origin, illuminated by an electromagnetic wave propagating along *z*-axis, with electric field polarized in *x*-axis. The sketch is shown in Fig. 1(a). Fig. 1(b) presents the four equivalent dipoles.

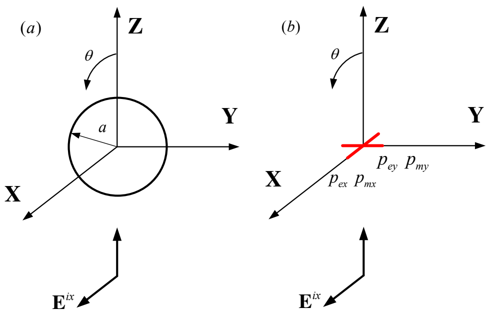

Fig. 1. (a) Electromagnetic scattering from a chiral sphere; (b) Equivalent Hertzian dipoles.

p_{ex}, p_{ey}, p_{mx} and p_{my} in Fig. 1(b) denotes, respectively, the equivalent Hertzian electric dipole moment along x-axis, Hertzian electric dipole moment along y-axis, Hertzian magnetic dipole moment along x-axis, and Hertzian magnetic dipole moment along y-axis. The scattered fields of the chiral sphere can be calculated by the total radiation fields of the four dipoles. The radiation problem of a Hertzian dipole can be solved by using dyadic Green functions [5]. With the help of the transformation of coordinate system and applying the principle of duality for dipoles, the radiation fields of the four dipoles can be readily obtained, in the following forms:

$$\begin{aligned} \mathbf{E}_{ex}^{s}(\mathbf{r}) &= -\frac{i\omega\mu e^{ikr}}{4\pi r} p_{ex}\left\{\hat{r}\left[\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]2\sin\theta\cos\phi+\left(-\cos\theta\cos\phi\hat{\theta}+\sin\phi\hat{\phi}\right)\left[1+\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]\right\}, \\ \mathbf{H}_{ex}^{s}(\mathbf{r}) &= ik\frac{e^{ikr}}{4\pi r} p_{ex}\left(1+\frac{i}{kr}\right)\left(\sin\phi\hat{\theta}+\cos\theta\cos\phi\hat{\phi}\right) \end{aligned} \tag{2}$$

$$\begin{aligned} \mathbf{E}_{ey}^{s}(\mathbf{r}) &= -\frac{i\omega\mu e^{ikr}}{4\pi r} p_{ey}\left\{\hat{r}\left[\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]2\sin\theta\sin\phi+\left(-\cos\theta\sin\phi\hat{\theta}-\cos\phi\hat{\phi}\right)\left[1+\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]\right\}, \\ \mathbf{H}_{ey}^{s}(\mathbf{r}) &= ik\frac{e^{ikr}}{4\pi r} p_{ey}\left(1+\frac{i}{kr}\right)\left(-\cos\phi\hat{\theta}+\cos\theta\sin\phi\hat{\phi}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \mathbf{E}_{mx}^{s}(\mathbf{r}) &= i\omega\mu\frac{e^{ikr}}{4\pi r} p_{mx}\left(1+\frac{i}{kr}\right)\left(\sin\phi\hat{\theta}+\cos\theta\cos\phi\hat{\phi}\right) \\ \mathbf{H}_{mx}^{s}(\mathbf{r}) &= \frac{ike^{ikr}}{4\pi r} p_{mx}\left\{\hat{r}\left[\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]2\sin\theta\cos\phi+\left(-\cos\theta\cos\phi\hat{\theta}+\sin\phi\hat{\phi}\right)\left[1+\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]\right\}, \end{aligned} \tag{4}$$

$$\begin{aligned} \mathbf{E}_{my}^{s}(\mathbf{r}) &= i\omega\mu\frac{e^{ikr}}{4\pi r} p_{my}\left(1+\frac{i}{kr}\right)\left(-\cos\phi\hat{\theta}+\cos\theta\sin\phi\hat{\phi}\right) \\ \mathbf{H}_{my}^{s}(\mathbf{r}) &= \frac{ike^{ikr}}{4\pi r} p_{my}\left\{\hat{r}\left[\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]2\sin\theta\sin\phi+\left(-\cos\theta\sin\phi\hat{\theta}-\cos\phi\hat{\phi}\right)\left[1+\frac{i}{kr}+\left(\frac{i}{kr}\right)^2\right]\right\}, \end{aligned} \tag{5}$$

where ω denotes the angular frequency of the Hertzian dipole, i.e. the angular frequency of the radiation wave; $k=\omega\sqrt{\varepsilon\mu}$ is the wave number of the radiation wave; ε, μ is the permittivity and the permeability of the surrounding medium. In the far field region, $kr \gg 1$, thus the term $1/kr$ in above expressions almost vanishes. While in the near field, $kr \ll 1$, only the term $(1/kr)^2$ in the expressions remains. Therefore, the near fields of the four dipoles become:

$$\mathbf{E}_{exNearField}^{s}(\mathbf{r}) = E_{ex}^{s}\frac{a^3}{r^3}\left[\hat{r}2\sin\theta\cos\phi+\left(-\cos\theta\cos\phi\hat{\theta}+\sin\phi\hat{\phi}\right)\right] \tag{6}$$

$$\mathbf{E}_{eyNearField}^{s}(\mathbf{r}) = E_{ey}^{s}\frac{a^3}{r^3}\left[\hat{r}2\sin\theta\sin\phi+\left(-\cos\theta\sin\phi\hat{\theta}-\cos\phi\hat{\phi}\right)\right] \tag{7}$$

$$\mathbf{H}_{mxNearField}^{s}(\mathbf{r}) = H_{mx}^{s}\frac{a^3}{r^3}\left[\hat{r}2\sin\theta\cos\phi+\left(-\cos\theta\cos\phi\hat{\theta}+\sin\phi\hat{\phi}\right)\right] \tag{8}$$

$$\mathbf{H}_{myNearField}^{s}(\mathbf{r}) = H_{my}^{s}\frac{a^3}{r^3}\left[\hat{r}2\sin\theta\sin\phi+\left(-\cos\theta\sin\phi\hat{\theta}-\cos\phi\hat{\phi}\right)\right] \tag{9}$$

where

$$E_{ex}^{s}=\frac{i\eta}{4\pi ka^{3}}p_{ex}\,,\ E_{ey}^{s}=\frac{i\eta}{4\pi ka^{3}}p_{ey}\,,\ H_{mx}^{s}=-\frac{i}{4\pi ka^{3}}p_{mx}\,,\ H_{my}^{s}=-\frac{i}{4\pi ka^{3}}p_{my}\,. \tag{10}$$

In the above expressions, $\eta=\sqrt{\mu/\varepsilon}$ is the wave impendence of the surround medium. a occurs here for the purpose of derivation of equivalent dipole moments in the following section. Note that in the near field, electric fields are mainly generated by electric dipoles and magnetic fields are mainly generated by magnetic dipoles.

2.2. Dipoles moments

The scattered fields of the Rayleigh chiral sphere can be equivalent to the radiation fields of the dipoles. Therefore the problem can be solved by applying the boundary conditions at the sphere surface. Taking account of constitutive relations of chiral media, electric field and magnetic field inside the chiral sphere are supposed to be:

$$\begin{aligned}\mathbf{E}&=E_x\hat{x}+E_y\hat{y}\\&=\hat{r}\left(E_x\sin\theta\cos\phi+E_y\sin\theta\sin\phi\right)+\hat{\theta}\left(E_x\cos\theta\cos\phi+E_y\cos\theta\sin\phi\right)+\hat{\phi}\left(-E_x\sin\phi+E_y\cos\phi\right),\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{H}&=H_x\hat{x}+H_y\hat{y}\\&=\hat{r}\left(H_x\sin\theta\cos\phi+H_y\sin\theta\sin\phi\right)+\hat{\theta}\left(H_x\cos\theta\cos\phi+H_y\cos\theta\sin\phi\right)+\hat{\phi}\left(-H_x\sin\phi+H_y\cos\phi\right).\end{aligned} \tag{12}$$

Thus, the electric displacement $\mathbf{D}$ and magnetic induction $\mathbf{B}$ can be expressed by using Eq. (1). And incident wave can be expressed as:

$$\mathbf{E}^{ix}=E_0e^{ikz}\hat{x}=E_0e^{ikz}\left(\hat{r}\sin\theta\cos\phi+\hat{\theta}\cos\theta\cos\phi-\hat{\phi}\sin\phi\right) \tag{13}$$

$$\mathbf{H}^{ix}=\frac{E_0}{i\omega\mu}ike^{ikz}\hat{y}=H_0e^{ikz}\left(\hat{r}\sin\theta\sin\phi+\hat{\theta}\cos\theta\sin\phi+\hat{\phi}\cos\phi\right) \tag{14}$$

At the surface of the sphere $r=a$, as $a\ll\lambda$, the scattered fields can adopt the near field expressions of the dipoles:

$$\mathbf{E}^{s}=\mathbf{E}_{exNearField}^{s}+\mathbf{E}_{eyNearField}^{s}\,,\ \mathbf{H}^{s}=\mathbf{H}_{mxNearField}^{s}+H_{myNearField}^{s} \tag{15}$$

The external fields contain incident field and scattered fields. According to the boundary conditions of Maxwell equations, the tangential components of the electric field and magnetic field continue at the surface; and the normal components of the electric displacement and magnetic induction continue at the surface. Therefore, we have the following equations:

$$-E_0+E_{ex}^{s}=-E_x \tag{16}$$

$$-E_{ey}^{s}=E_y \tag{17}$$

$$H_0-H_{my}^{s}=H_y \tag{18}$$

$$H_{mx}^{s}=-H_x \tag{19}$$

$$\varepsilon E_0+\varepsilon E_{ex}^{s}2=\varepsilon_cE_x+i\kappa\sqrt{\varepsilon_0\mu_0}H_x \tag{20}$$

$$\varepsilon E_{ey}^{s}2=\varepsilon_cE_y+i\kappa\sqrt{\varepsilon_0\mu_0}H_y \tag{21}$$

$$2\mu H_{mx}^{s}=-i\kappa\sqrt{\varepsilon_0\mu_0}E_x+\mu_cH_x \tag{22}$$

$$\mu H_0+2\mu H_{my}^{s}=-i\kappa\sqrt{\varepsilon_0\mu_0}E_y+\mu_cH_y \tag{23}$$

The unknowns about the radiation fields $E_{ex}^s, E_{ey}^s, H_{my}^s, H_{mx}^s$, and the unknowns about the internal fields E_x, E_y, H_x, H_y can all be solved from the above equations. With the relations in Eq. (10), the four equivalent dipole moments for the chiral sphere can be solved as follows:

$$p_{ex} = -4\pi ika^3 \sqrt{\frac{\varepsilon}{\mu}} \frac{(\varepsilon_c - \varepsilon)(2\mu + \mu_c) - \kappa^2 \varepsilon_0 \mu_0}{(2\varepsilon + \varepsilon_c)(2\mu + \mu_c) - \kappa^2 \varepsilon_0 \mu_0} E_0 \tag{24}$$

$$p_{ey} = 4\pi ka^3 \frac{3\varepsilon\kappa\sqrt{\varepsilon_0 \mu_0}}{(2\varepsilon + \varepsilon_c)(2\mu + \mu_c) - \kappa^2 \varepsilon_0 \mu_0} E_0 \tag{25}$$

$$p_{mx} = p_{ey} \tag{26}$$

$$p_{my} = 4\pi ika^3 \sqrt{\frac{\varepsilon}{\mu}} \frac{(2\varepsilon + \varepsilon_c)(\mu_c - \mu) - \kappa^2 \varepsilon_0 \mu_0}{(2\varepsilon + \varepsilon_c)(2\mu + \mu_c) - \kappa^2 \varepsilon_0 \mu_0} E_0 \tag{27}$$

Eqs. (24)-(27) are consistent with those presented in Ref. [4] by using quasi-static analysis. It can be seen that the equivalent dipole moments are related to the permittivity and permeability of the surrounding medium and the chiral sphere, the chirality parameter of the sphere, the radius of the sphere, and wave number of the incident wave. For $\kappa = 0$, p_{ey} and p_{mx} vanish; and the model degenerates to an isotropic sphere case.

3. Numerical results and discussions

According to Eqs. (2)-(5), the radiation fields of the four dipoles can be calculated. The scattered fields of the chiral sphere are the superposition of the radiation fields:

$$\begin{aligned} \mathbf{E}^s &= \mathbf{E}_{ex}^s + \mathbf{E}_{ey}^s + \mathbf{E}_{mx}^s + \mathbf{E}_{my}^s \\ \mathbf{H}^s &= \mathbf{H}_{ex}^s + \mathbf{H}_{ey}^s + \mathbf{H}_{mx}^s + \mathbf{H}_{my}^s \end{aligned}. \tag{28}$$

In the far field region, as stated in section 2.1, terms with $1/kr$ almost vanish; and the radiation field corresponding to p_{ex} becomes:

$$\begin{aligned} \mathbf{E}_{exFarField}^s(\mathbf{r}) &= -i\omega\mu p_{ex} \frac{e^{ikr}}{4\pi r}\left(-\cos\theta\cos\phi\hat{\theta} + \sin\phi\hat{\phi}\right) \\ \mathbf{H}_{exFarField}^s(\mathbf{r}) &= ikp_{ex} \frac{e^{ikr}}{4\pi r}\left(\sin\phi\hat{\theta} + \cos\theta\cos\phi\hat{\phi}\right) \end{aligned}. \tag{29}$$

The other radiation fields $\mathbf{E}_{ey}^s, \mathbf{H}_{ey}^s, \mathbf{E}_{mx}^s, \mathbf{H}_{mx}^s, \mathbf{E}_{my}^s, \mathbf{H}_{my}^s$ will be in similar forms.

Stokes parameters I_s and V_s are calculated to show the far field scattered fields of the chiral sphere:

$$I_s = E_{\parallel}^s E_{\parallel}^{s*} + E_{\perp}^s E_{\perp}^{s*}, V_s = i(E_{\parallel}^s E_{\perp}^{s*} - E_{\perp}^s E_{\parallel}^{s*}),, \tag{30}$$

where $E_{\parallel}^s = E_\theta^s$, $E_{\perp}^s = -E_\phi^s$. The parameter I_s presents the intensity of the scattered field; and V_s/I_s estimates the polarization state of the field. $V_s/I_s < 0$ suggests that the wave is right-handed polarized, where $V_s/I_s = -1$ indicates a right-handed circularly polarized (RCP) wave. Similarly, $V_s/I_s > 0$ suggests a left-handed polarized wave and $V_s/I_s = 1$ means that the wave is left-handed circularly polarized (LCP).

Scattering patterns of a small sized chiral sphere with different chirality parameters are depicted in Fig. 2. Note that the colour in the figure is specified by value of V_s/I_s. Therefore, as shown in the figures, a linearly polarized wave is depicted in green; a RCP wave is depicted in blue and a LCP wave is depicted in red. The parameters of the sphere are: $\varepsilon_c = 1.7689\varepsilon_0, \mu_c = 1.0\mu_0, \lambda = 0.488, a = 0.001$. And the chirality parameter in Fig. 2(a), Fig. 2(b), Fig. 2(c) and Fig. 2(d) is 0.0, 0.2, 0.4, and 0.6, respectively. For $\kappa = 0$, the chiral sphere degenerates to an isotropic sphere. As $\mu_c = 1.0\mu_0$, only p_{ex} among the four dipole moments is nonzero, which means that the scattering pattern is just the radiation pattern of a Hertzian electric dipole along *x*-axis. And it can be seen from Fig. 2(a) that the scattered field is linearly polarized. Fig. 2(b), Fig. 2(c), and Fig. 2(d) show the effects of the chirality parameter. It can be seen that the far field scattered fields of a chiral sphere are not linearly polarized any more. Chirality changes both the polarization state and the angular distributions of the scattering intensity.

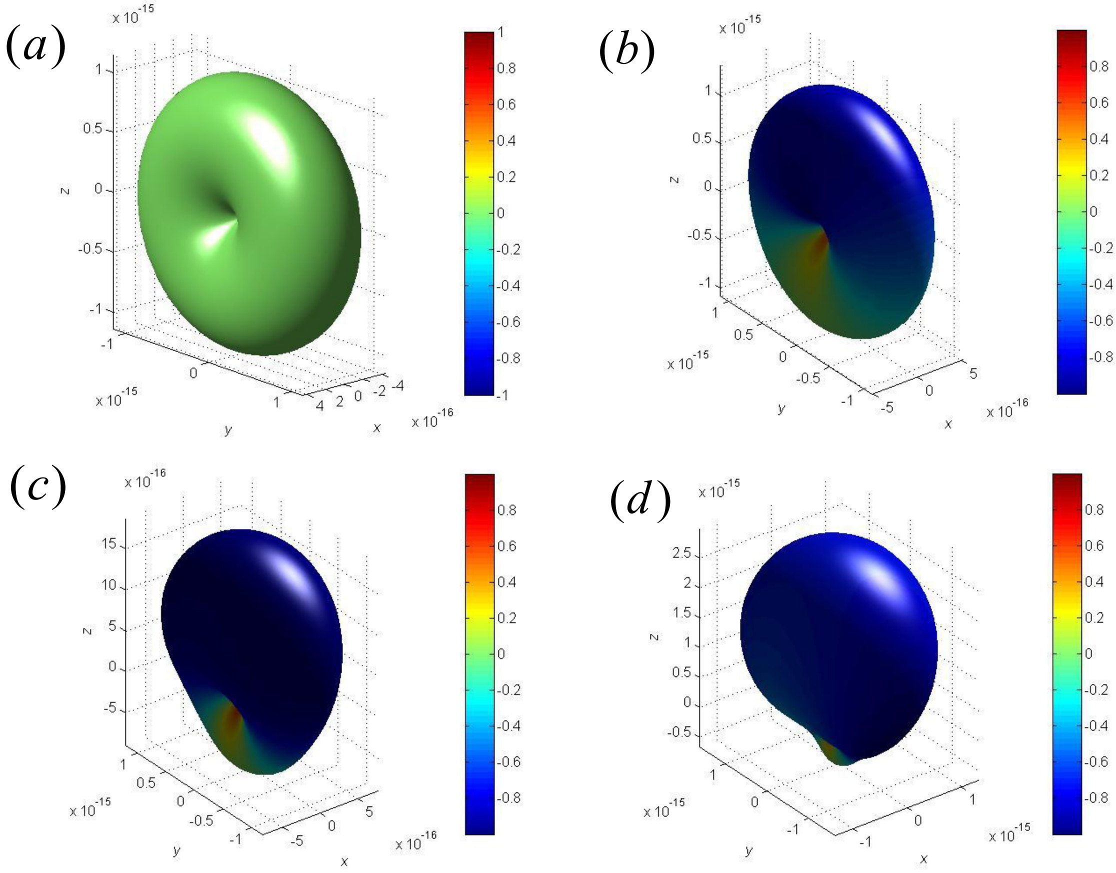

Fig. 2. Scattering patterns of a chiral sphere. (a) $\kappa = 0$; (b) $\kappa = 0.2$; (c) $\kappa = 0.4$; (d) $\kappa = 0.6$.
$\varepsilon_c = 1.7689\varepsilon_0, \mu_c = 1.0\mu_0, \lambda = 0.488, a = 0.001.$

4. Conclusions

In this paper, problems of Rayleigh scattering of chiral spheres are converted to radiation problems of dipoles. The equivalent dipole moments can be determined by applying the boundary conditions of Maxwell equations. It concludes that the dipole moments are associated with the parameters of the chiral sphere and incident wave. The far field scattered fields of the chiral sphere are depicted and it is found that chirality changes both the polarization and the intensity of the scattered fields. The model can be readily extended to the scattering problem of multi-chiral particles with Rayleigh sizes. And the method may offer new ways to analyze the optics characteristic of materials with periodic chiral microstructures.

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